Resolução - Tarefa 22 – Frente B – Professor Rogério
Transformações trigonométricas - COMPLEMENTARES
Adição e subtração de arcos seno, cosseno e tangente

- $\text{sen}(a+b) = \text{sen } a \cdot \cos b + \text{sen } b \cdot \cos a$
- $\text{sen}(a-b) = \text{sen } a \cdot \cos b - \text{sen } b \cdot \cos a$
- $\cos(a+b) = \cos a \cdot \cos b - \text{sen } a \cdot \text{sen } b$
- $\cos(a-b) = \cos a \cdot \cos b + \text{sen } a \cdot \text{sen } b$

$$\text{tg}(a+b) = \frac{\text{tg } a + \text{tg } b}{1 - \text{tg } a \cdot \text{tg } b} \qquad \text{tg}(a-b) = \frac{\text{tg } a - \text{tg } b}{1 + \text{tg } a \cdot \text{tg } b}$$

01.

a) $\text{sen}25^\circ.\cos 20^\circ + \text{sen}20^\circ.\cos 25^\circ =$

$= \text{sen}(25^\circ + 20^\circ) = \text{sen}45^\circ = \frac{\sqrt{2}}{2}$

b) $\cos 70^\circ.\cos 10^\circ + \text{sen}70^\circ.\text{sen}10^\circ = \cos(70^\circ - 10^\circ) = \cos 60^\circ = \frac{1}{2}$

02.

$$\text{tg}(x+y) = \frac{\text{tg}x + \text{tg}y}{1 - \text{tg}x.\text{tg}y} \rightarrow 33 = \frac{\text{tg}x + 3}{1 - \text{tg}x.3} \rightarrow$$

$$\rightarrow \text{tg}x + 3 = 33 - 99\text{tg}x \rightarrow 100\text{tg}x = 30 \rightarrow \text{tg}x = \frac{3}{10}$$

03. $\text{tg}(\alpha+\beta) = \frac{5}{20} \rightarrow \text{tg}(\alpha+\beta) = \frac{1}{4}$

$\text{tg}\beta = \frac{1}{20}$

$$\text{tg}(\alpha+\beta) = \frac{\text{tg}\alpha + \text{tg}\beta}{1 - \text{tg}\alpha.\text{tg}\beta} \rightarrow \frac{\text{tg}\alpha + \frac{1}{20}}{1 - \text{tg}\alpha.\frac{1}{20}} = \frac{1}{4} \rightarrow$$

$$\rightarrow \text{tg}\alpha + \frac{1}{20} = \frac{1}{4} - \frac{1}{80}.\text{tg}\alpha \Rightarrow \text{tg}\alpha + \frac{1}{80}\text{tg}\alpha = \frac{1}{4} - \frac{1}{20}$$

$$\frac{81}{80}\text{tg}\alpha = \frac{4}{20} \rightarrow \text{tg}\alpha = \frac{\frac{1}{5}}{\frac{81}{80}} \rightarrow \text{tg}\alpha = \frac{16}{81}$$

04. c

$\text{sen}(\alpha+\beta) + \text{sen}(\alpha-\beta) = m$

$\text{sen}\alpha.\cos\beta + \text{sen}\beta.\cos\alpha + \text{sen}\alpha.\cos\beta - \text{sen}\beta.\cos\alpha = m \rightarrow$

$\rightarrow 2\text{sen}\alpha.\cos\beta = m$

$\cos(\alpha+\beta) + \cos(\alpha-\beta) = n$

$\cos\alpha.\cos\beta - \text{sen}\alpha.\text{sen}\beta + \cos\alpha.\cos\beta + \text{sen}\alpha.\text{sen}\beta = n \rightarrow$

$\rightarrow 2\cos\alpha.\cos\beta = n$

$$\frac{m}{n} = \frac{2\text{sen}\alpha.\cos\beta}{2\cos\alpha.\cos\beta} \rightarrow \frac{m}{n} = \frac{\text{sen}\alpha}{\cos\alpha} \rightarrow \frac{m}{n} = \text{tg}\alpha$$

05. d

$x - y = 60^\circ \rightarrow y = 60^\circ - x$

$\cos(x+y)^2 + \text{sen}(x+y)^2$

$\cos^2 x + 2\cos x.\cos y + \cos^2 y + \text{sen}^2 x + 2\text{sen}x.\text{sen}y + \text{sen}^2 y$

$(\cos^2 x + \text{sen}^2 x) + (\cos^2 y + \text{sen}^2 y) + 2.\cos x.\cos y + 2\text{sen}x.\text{sen}y$

$1 + 1 + 2(\cos x.\cos y + \text{sen}x.\text{sen}y)$

$2 + 2.\cos(x-y)$

$2 + 2.\cos 60^\circ$

$2 + 2.\frac{1}{2} = 3$

07.

a) $\text{sen}15^\circ = \text{sen}(45^\circ - 30^\circ) = \text{sen}45^\circ.\cos 30^\circ - \text{sen}30^\circ.\cos 45^\circ \rightarrow$

$$\rightarrow \text{sen}15^\circ = \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{\sqrt{2}}{2} \rightarrow \text{sen}15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$$

b)

$$A = \frac{1.1.\text{sen}15^\circ}{2} \rightarrow A = \frac{1.1.\frac{\sqrt{6}-\sqrt{2}}{4}}{2} \rightarrow$$

$$\rightarrow A = 24.\frac{\sqrt{6}-\sqrt{2}}{8} \rightarrow A = 3(\sqrt{6}-\sqrt{2})$$