UFU

AULA DE VÉSPERA VESTIBULAR 2018

MATEMÁTICA

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Prof. Luiz Henrique

Prof. Diego Bernadelli

Estatística de Conteúdos Abordados MATEMÁTICA - UFU

Conteúdo	2000 - 2016	2017
Álgebra	3	
Análise Combinatória	7	1
Conjuntos	6	
Determinantes	3	
Equações Polinomiais	4	
Estatística	5	1
Exponenciais	2	
Funções	20	1
Geometria Analítica	12	
Geometria Espacial	14	2
Geometria Plana	12	2
Grandezas Proporcionais	5	
Logaritmos	4	1
Matrizes	3	
Números Complexos	2	
Polinômios	4	1
Probabilidades	9	
Progressão Aritmética	5	
Progressão Geométrica	2	1
Sistemas Lineares	7	
Trigonometria	9	1
Total	138	11

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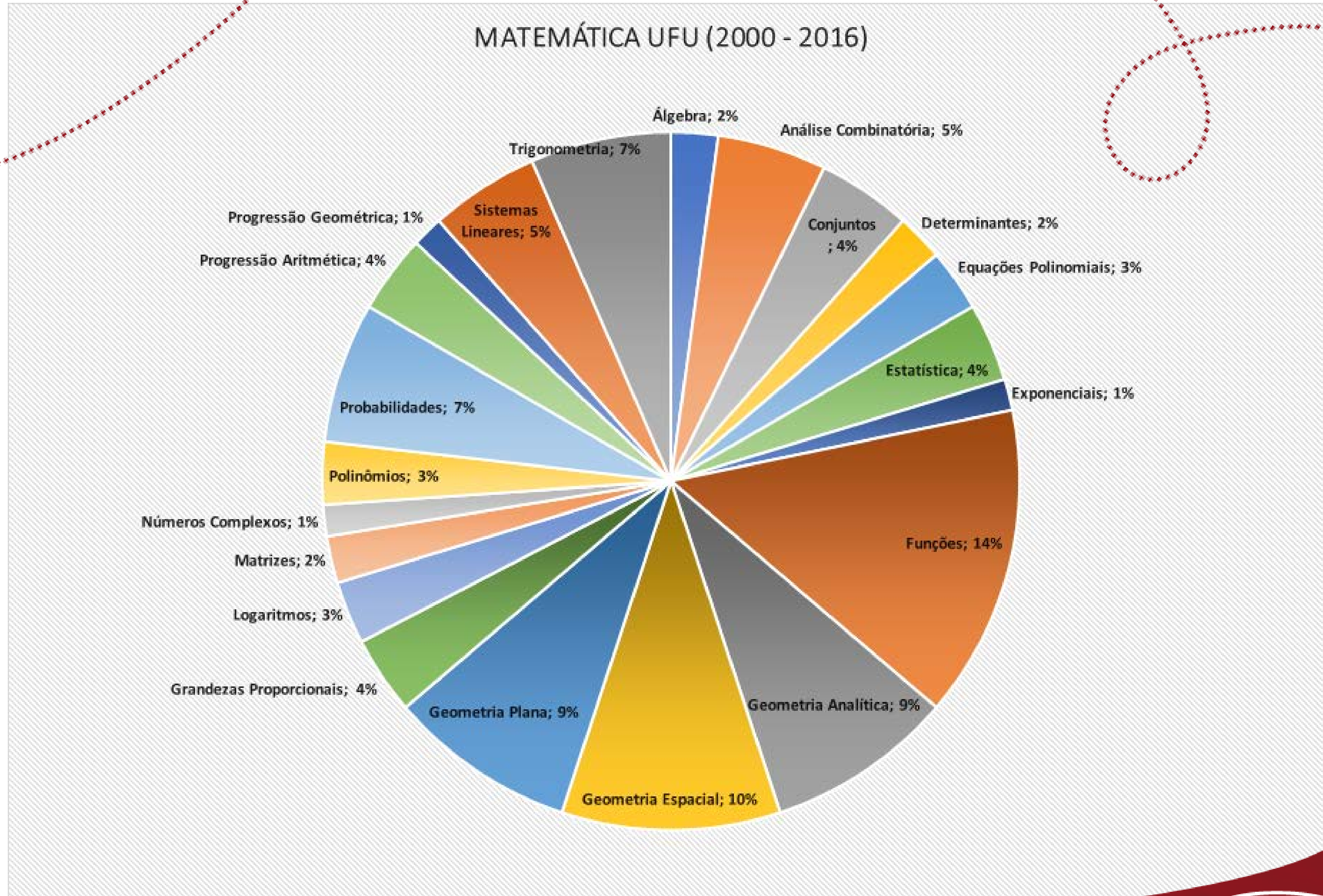

MATEMÁTICA UFU (2000 - 2016)
Álgebra; 2%
Análise Combinatória; 5%
Conjuntos ; 4%
Determinantes; 2%
Equações Polinomiais; 3%
Estatística; 4%
Exponenciais; 1%
Funções; 14%
Geometria Analítica; 9%
Geometria Espacial; 10%
Geometria Plana; 9%
Grandezas Proporcionais; 4%
Logaritmos; 3%
Matrizes; 2%
Números Complexos; 1%
Polinômios; 3%
Probabilidades; 7%
Progressão Aritmética; 4%
Progressão Geométrica; 1%
Sistemas Lineares; 5%
Trigonometria; 7%

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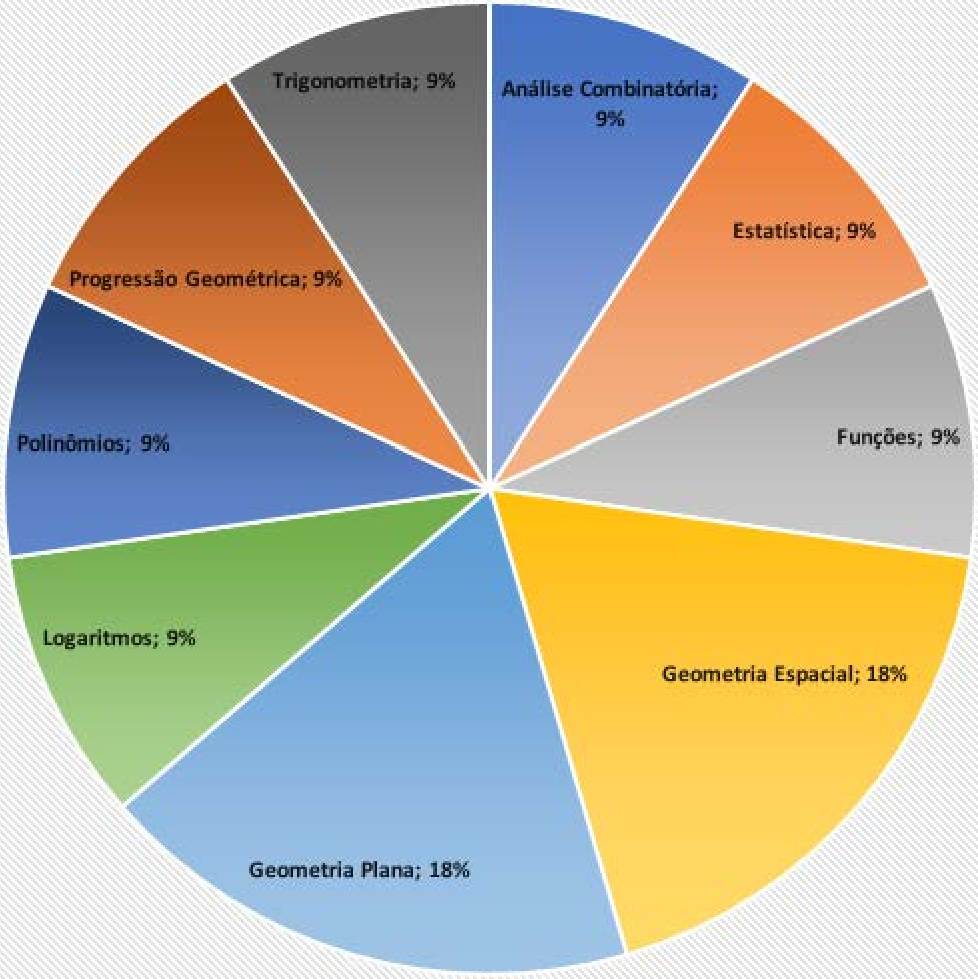
MATEMÁTICA UFU (2017)



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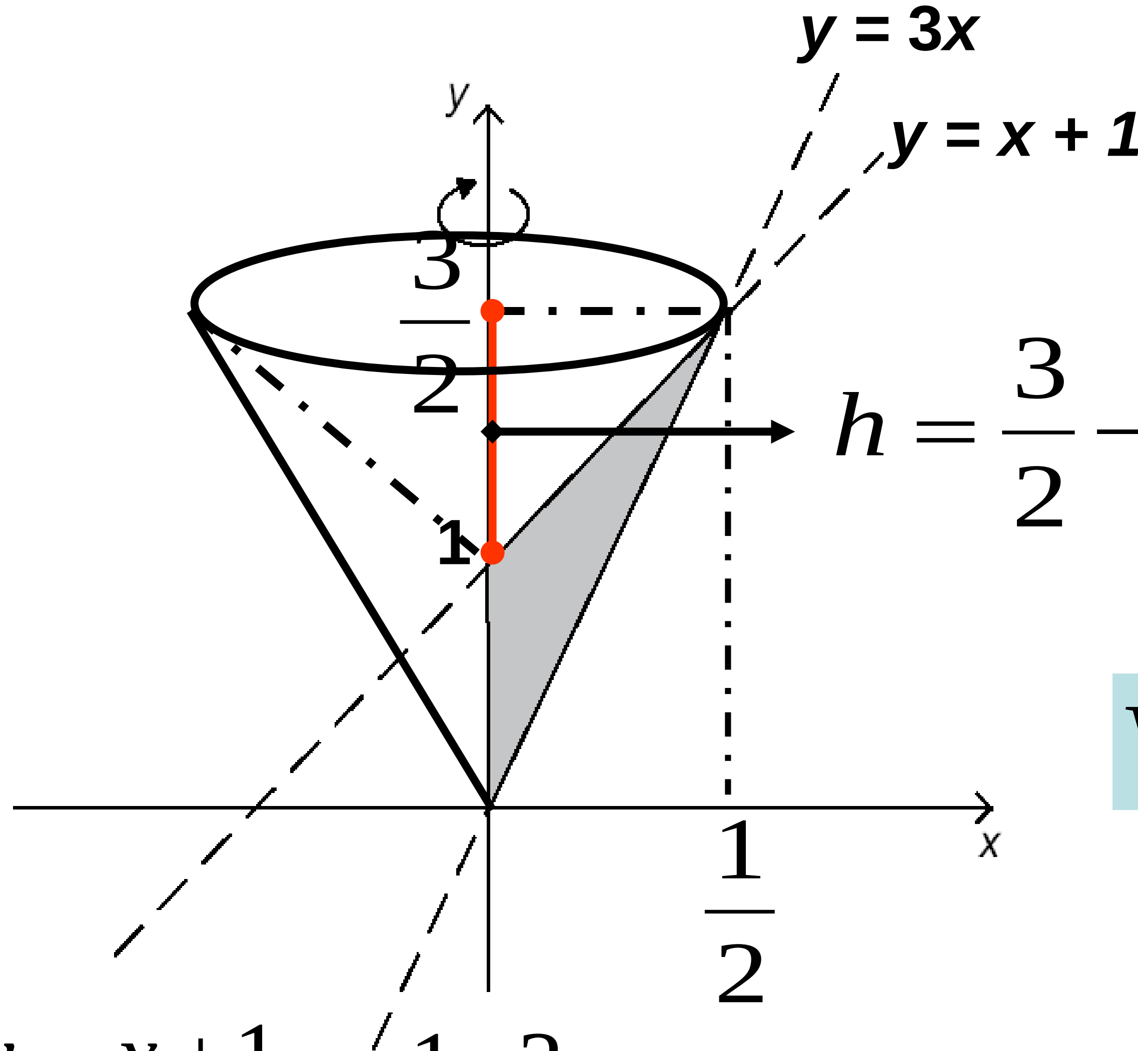
01)



$$h = \frac{3}{2} - 1 = \frac{1}{2}$$

$$V_F = V_2 - V_1$$

$$\begin{cases} y = x + 1 \\ y = 3x \end{cases} \Rightarrow \left(\frac{1}{2}; \frac{3}{2}\right)$$

$$V_F = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2 \cdot \frac{3}{2} - \frac{1}{3}\pi\left(\frac{1}{2}\right)^2 \cdot \frac{1}{2}$$

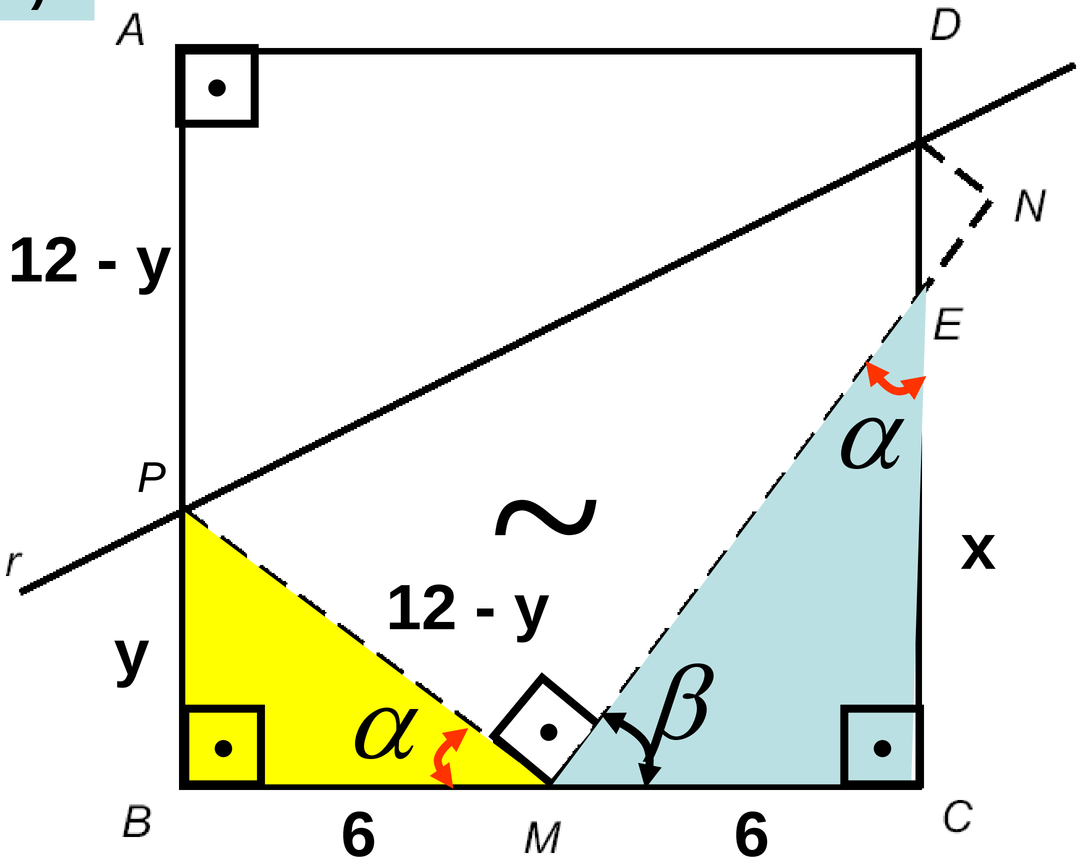
$$V_F = \frac{1}{3}\pi\frac{1}{4} \cdot \frac{3}{2} - \frac{1}{3}\pi\frac{1}{4} \cdot \frac{1}{2}$$

$$V_F = \frac{3\pi}{24} - \frac{\pi}{24}$$

$$V_F = \frac{2\pi}{24} \Rightarrow V_F = \frac{\pi}{12}(u.c)^3$$

A

02)



y = 9/2

$$\frac{x}{6} = \frac{6}{y}$$

$$x.\frac{9}{2} = 36$$

4

$$x = 8cm$$

$(12 – y)^2 = y^2 + 6^2$

$144 – 24y + y^2 = y^2 + 36$

C

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03)

$$P(a;b) \in y = 3x$$

$$\mapsto a + b$$

$$(r)\,3x + 4y = 0$$

$$d_{(P;r)} = 3$$

$$i)\ b = 3a \Rightarrow P(a;3a)$$

$$ii)\ \frac{|3(a) + 4(3a)|}{\sqrt{3^2 + 4^2}} = 3$$

$$|15a| = 15$$

$$\bullet\ 15a = 15 \qquad \bullet\ 15a = -15$$

$$a = 1 \qquad a = -1$$

$$\therefore P(1;3)$$

$$a + b = 1 + 3$$

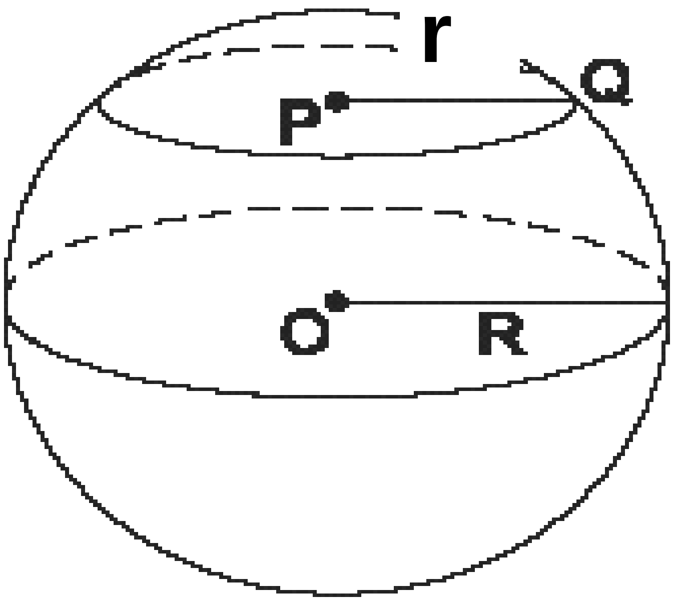
$$a + b = 4$$

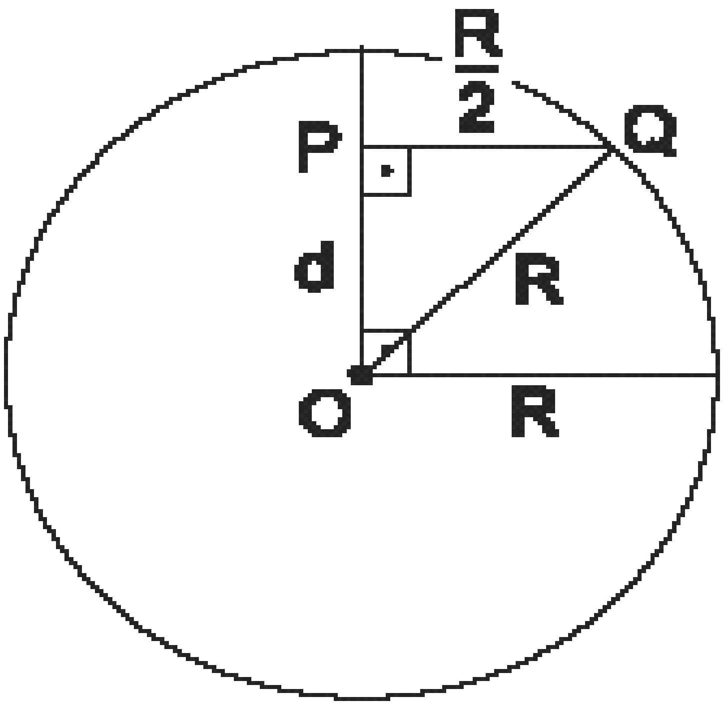
D

04)

Resolução:

$$\frac{\pi . r^2}{\pi . R^2} = \frac{1}{4} \Rightarrow \frac{r}{R} = \sqrt{\frac{1}{4}} \Rightarrow \frac{r}{R} = \frac{1}{2} \Rightarrow r = \frac{R}{2}$$





$$R^2 = d^2 + \left(\frac{R}{2}\right)^2 \Rightarrow R^2 = d^2 + \frac{R^2}{4}$$

$$R^2 = d^2 + \frac{R^2}{4}$$

$$4R^2 = 4d^2 + R^2$$

$$3R^2 = 4d^2$$

$$d^2 = \frac{3R^2}{4}$$

$$d = \sqrt{\frac{3R^2}{4}}$$

$$\therefore d = \frac{R\sqrt{3}}{2}$$

C

05)

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Termo geral:

$$(a-b)^n \longrightarrow T_{(p+1)} = \binom{n}{p} a^{n-p}.(-b)^p$$

$$(sen2x - 3\cos 2x)^4 \Rightarrow T_3 = \binom{4}{2}(sen2x)^2(-3\cos 2x)^2$$

$$T_1; T_2; T_3; T_4; T_5$$

$$T_3 = \frac{4!}{2!.2!}.(sen2x)^2.9(\cos 2x)^2$$

$$T_3 = 3.2.(sen2x)^2.9(\cos 2x)^2$$

$$T_3 = 27.2(sen2x)^2.(\cos 2x)^2$$

$$T_3 = \frac{27.2.2}{2}(sen2x)^2.(\cos 2x)^2$$

$$T_3 = \frac{27}{2}.2^2(sen2x)^2.(\cos 2x)^2$$

$$T_3 = \frac{27}{2}.(2.sen2x.\cos 2x)^2$$

$$T_3 = \frac{27}{2}.(sen4x)^2$$

D

$$2.T_3 = 27.(sen4x)^2$$

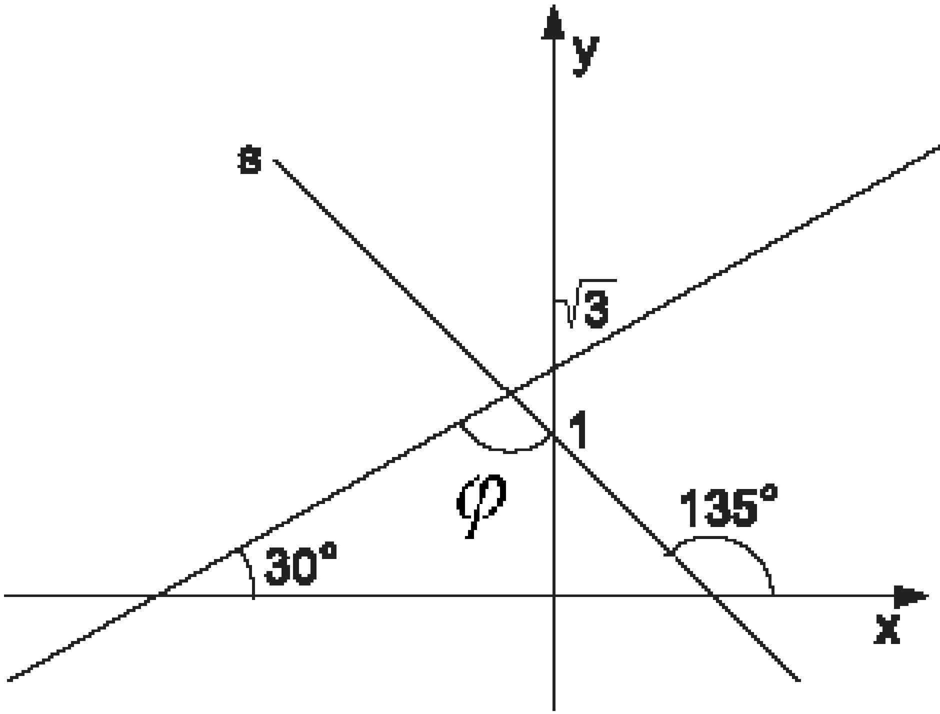
06)

$$r: \sqrt{3}\,y = x + 3 \Leftrightarrow y = \frac{\sqrt{3}}{3}x + \sqrt{3}$$

$$m_r = \frac{\sqrt{3}}{3} \therefore \alpha_r = 30^{\circ}$$

$$s: y = -x + 1$$

$$m_s = -1 \therefore \alpha$$



$$135^{\circ} = \varphi + 30^{\circ}$$

$$\varphi = 105^{\circ}$$

C

07)

$$7.sen(3x).\cos(2x)+7sen(2x).sen(\frac{\pi}{2}-3x)=1$$

$$sen(\frac{\pi}{2}-3x)=\cos(3x)$$

$$7.sen(3x).\cos(2x)+7sen(2x).\cos(3x)=1$$

$$sen(3x).\cos(2x)+sen(2x).\cos(3x)=\frac{1}{7}$$

$$sen(a+b)=sena.\cos b+senb.\cos a$$

$$sen(3x+2x)=\frac{1}{7} \Rightarrow sen(5x)=\frac{1}{7}$$

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$$sen(5x) = \frac{1}{7}$$

$$\mapsto \cos(10x) \; ou \; \cos(5x)$$

$$\cos(2a) = 1 - 2sen^2(a)$$

$$\cos(10x) = 1 - 2sen^2(5x)$$

$$\cos(10x) = 1 - 2.\left(\frac{1}{7}\right)^2$$

$$\cos(10x) = 1 - 2.\frac{1}{49} = \frac{49-2}{49}$$

$$\cos(10x) = \frac{47}{49}$$

D

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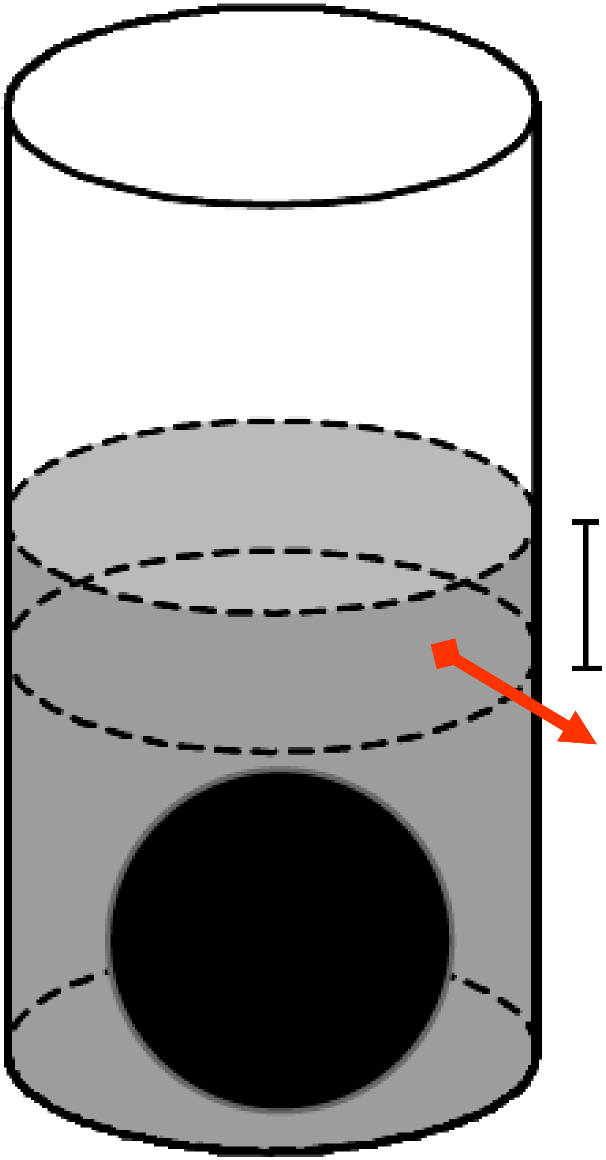
08)



$$V_E = V_{CILINDRO}$$

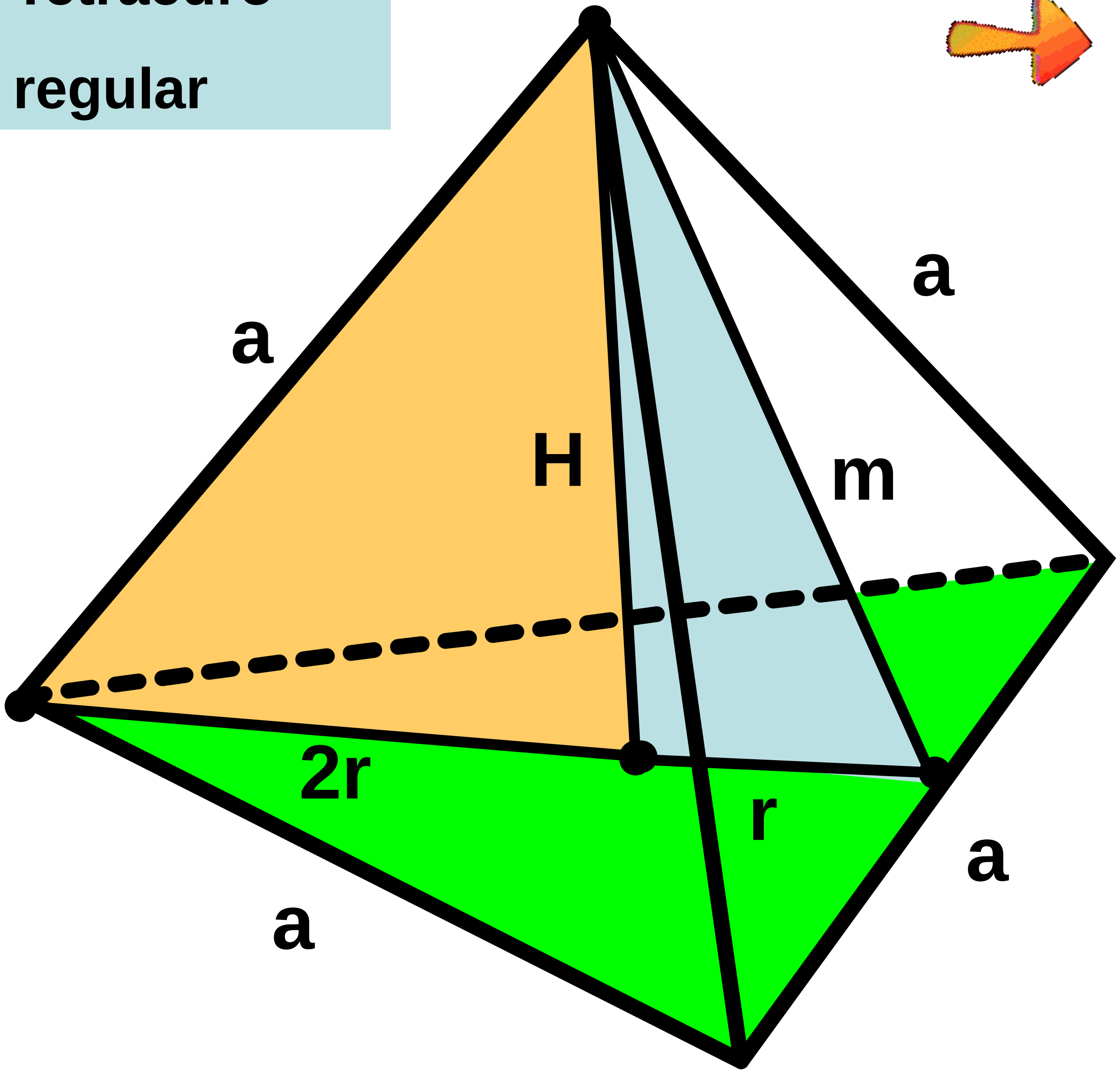
$$V_E = \frac{4}{3}\pi.r^3 = \pi.R^2.\frac{9R}{16}$$

$$\frac{4}{3}.r^3 = \frac{9R^3}{16}$$

$$r = \frac{3R}{4}cm$$

A

Tetraedro regular



$$\bullet V = \frac{a^3\sqrt{2}}{12}$$

U F U

$$\bullet S_T = 4.\frac{a^2\sqrt{3}}{4}$$

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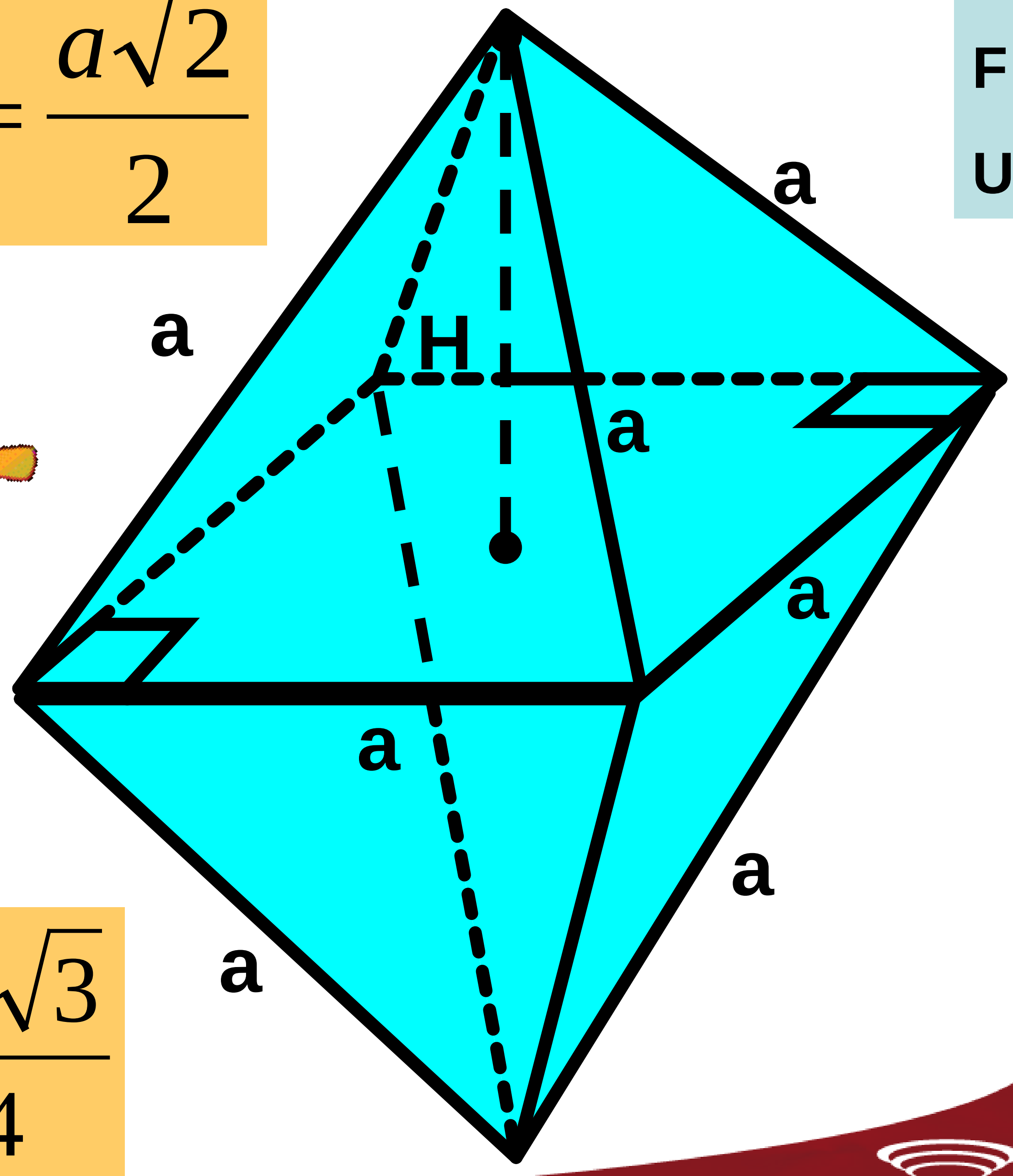
Octaedro regular

$$H = \frac{a\sqrt{2}}{2}$$

$$V = \frac{a^3.\sqrt{2}}{3}$$

- $S_T = 8.\frac{a^2\sqrt{3}}{4}$



U F U

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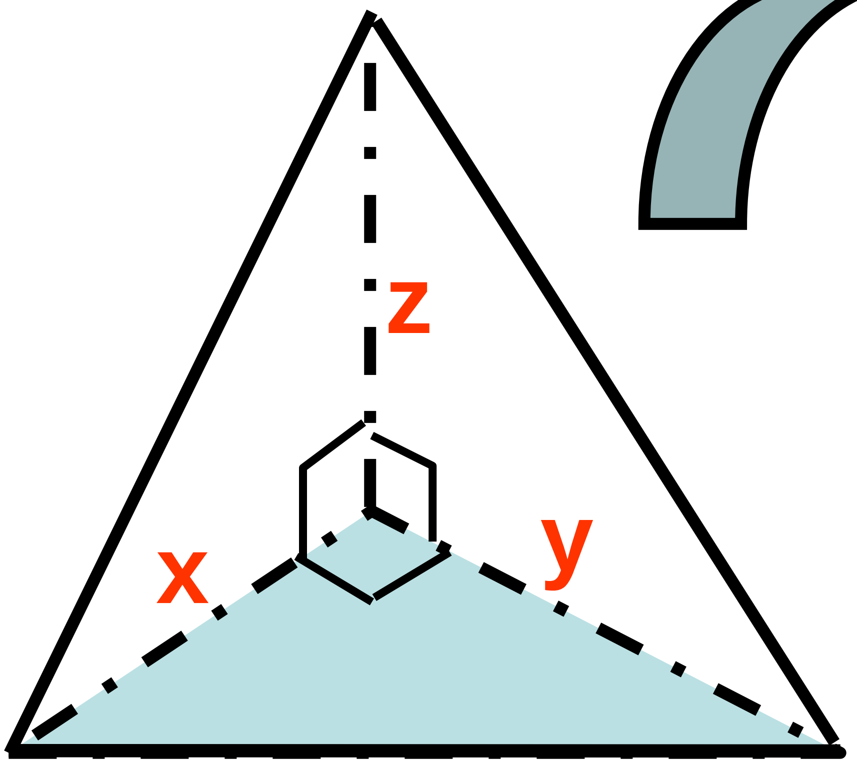
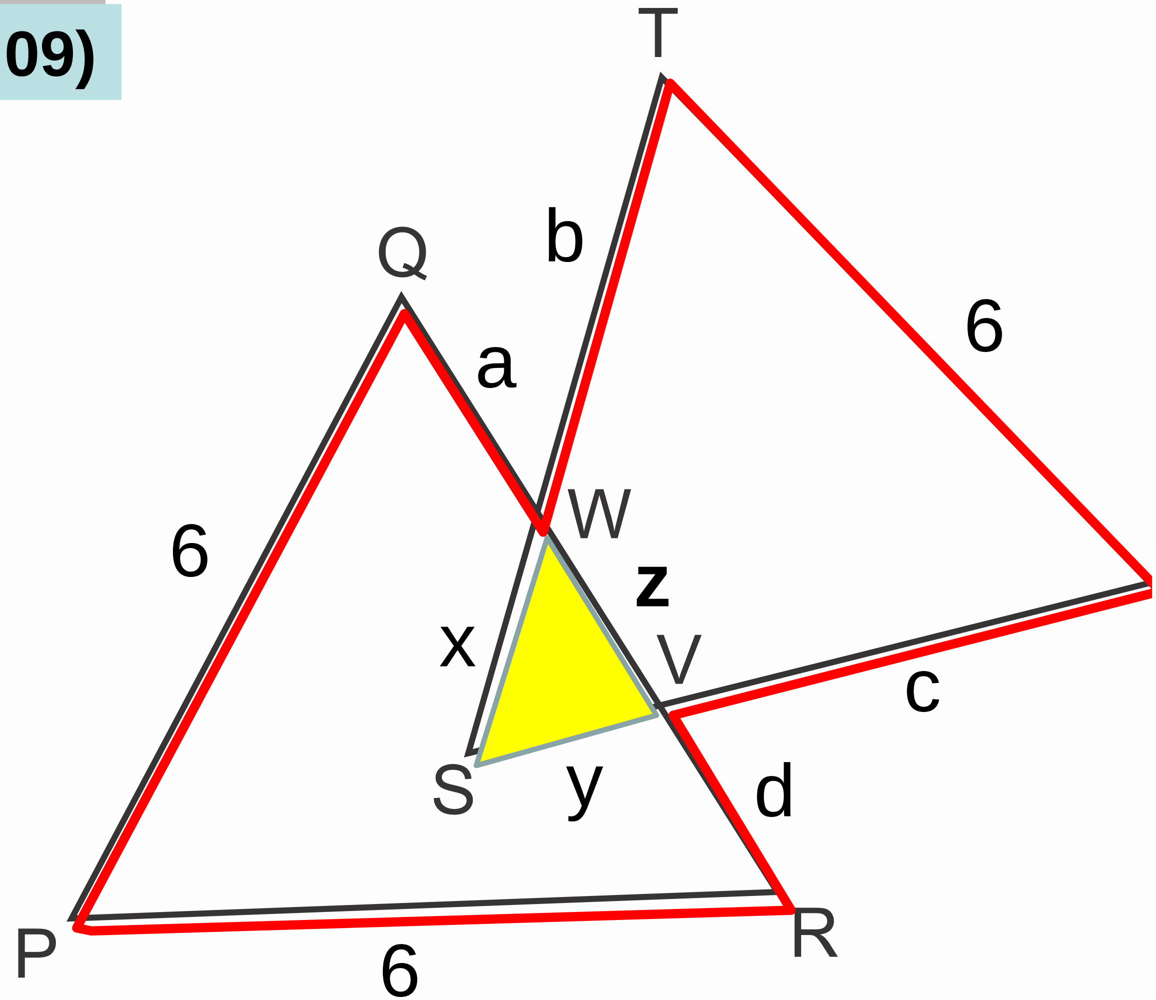
Tetraedro tri-retangular



$$V = \frac{x.y.z}{6}$$

$$V = \frac{1}{3} S_B . H = \frac{1}{3}\left(\frac{x.y}{2}\right).z = \frac{x.y.z}{6}$$

09)



$$(2)\ a+z+d=6$$

$$x+b=6$$

$$y+c+6$$

$$\mapsto 2p=a+b+c+d+6+6+6$$

$$(1)\ x+y+z=9$$

$$\mapsto 2p = a + b + c + d + 18$$

$$(1) \quad x + y + z = 9$$

$$2p = 9 + 18$$

$$(2) \quad a + z + d = 6$$

$$x + b = 6$$

$$y + c + 6$$

$$2p = 27cm$$

(+)

$$a + b + c + d + x + y + z = 18$$

$$a + b + c + d + 9 = 18$$

$$a + b + c + d = 9$$

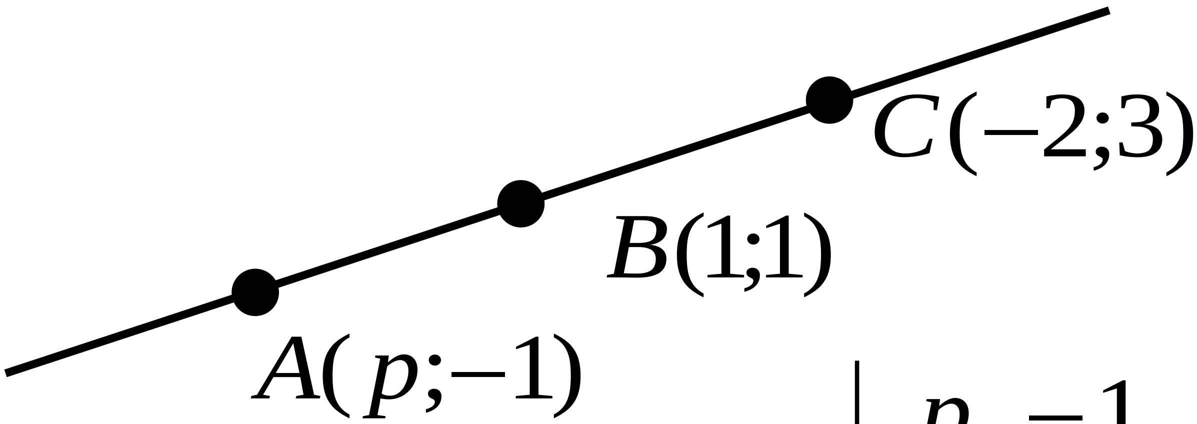
C

10)

$$x^2 + y^2 + 4x - 6y + 12 = 0$$

- $-2a = 4 \quad \therefore\ a = -2$

  $-2b = -6 \ \therefore\ b = 3$ $\Rightarrow$ $C(-2;3)$



$$\begin{vmatrix} p & -1 & 1 \\ 1 & 1 & 1 \\ -2 & 3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} p & -1 & 1 \\ 1 & 1 & 1 \\ -2 & 3 & 1 \end{vmatrix}\begin{matrix} p & -1 \\ 1 & 1 \\ -2 & 3 \end{matrix} = 0$$

$$p + 2 + 3 + 2 - 3p + 1 = 0$$

$$-2p + 8 = 0$$

$$2p = 8$$

$$\therefore p = 4$$

D

11) Dado:

$$f(xy) = -\frac{f(-y)}{x} \quad \text{e} \quad f(1) = 2, \quad \mapsto f\left(\frac{1}{2}\right)$$

$$f\left((-2)\cdot\left(-\frac{1}{2}\right)\right) = \frac{-f\left(\frac{1}{2}\right)}{-2}$$

$$f(1) = \frac{-f\left(\frac{1}{2}\right)}{-2}$$

$$2 = \frac{f\left(\frac{1}{2}\right)}{2}$$

B

12)

Utilizando a Regra de Cramer:

$$SI \;\text{ ou }\; SPI \Rightarrow D = 0$$

$$\begin{cases} x + ay + z = 2 \\ -x - 2y + 3z = -1 \\ 3x + 0y + az = 5 \end{cases} \Rightarrow D = \begin{vmatrix} 1 & a & 1 \\ -1 & -2 & 3 \\ 3 & 0 & a \end{vmatrix} = a^2 + 7a + 6 = 0 \Rightarrow \begin{cases} a' = -1 \\ a'' = -6 \end{cases}$$

$$x = \frac{D_x}{D} \Rightarrow D_x \neq 0$$

$$D_x = \begin{vmatrix} 2 & a & 1 \\ -1 & -2 & 3 \\ 5 & 0 & a \end{vmatrix} = a^2 + 11a + 10 \neq 0 \Rightarrow \begin{cases} a' \neq -1 \\ a'' \neq -10 \end{cases}$$

D

Assim, a = -6

13) $y = 82 - 12\log(t+1),$

Devemos usar $y = 70$

$$70 = 82 - 12\log(t+1) \Leftrightarrow 12\log(t+1) = 12$$

$$\log(t+1) = 1$$

$$t + 1 = 10^1$$

$$t = 9.$$

B

14) $\begin{cases} \dfrac{x}{y} + \dfrac{y}{z} = 6 \\ \dfrac{x}{y} + \dfrac{z}{x} = \dfrac{5}{2} \\ \dfrac{y}{z} + \dfrac{z}{x} = \dfrac{9}{2} \end{cases}$ (+)

$$\frac{x^2z + y^2x + z^2y}{xyz}$$

$$2.\frac{x}{y} + 2.\frac{y}{z} + 2.\frac{z}{x} = 13$$

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = \frac{13}{2}$$

$$\frac{x^2z + xy^2 + yz^2}{xyz} = \frac{13}{2}$$

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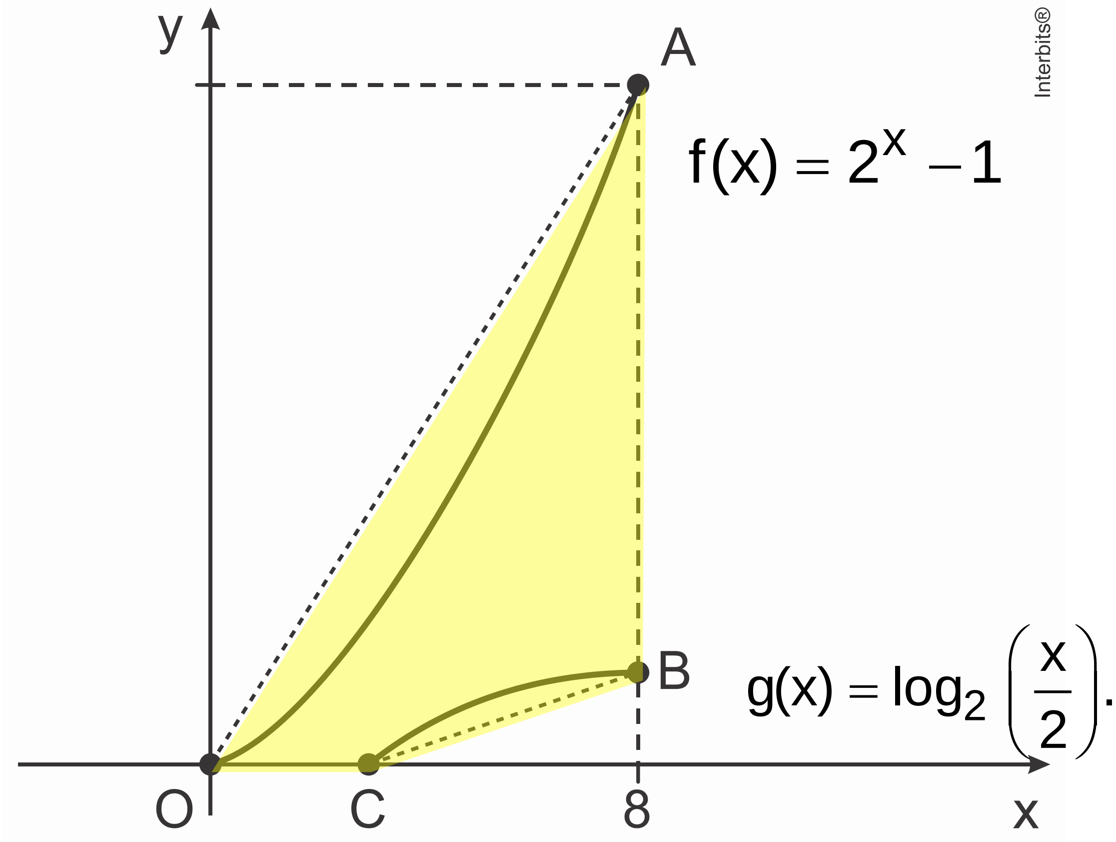
15)

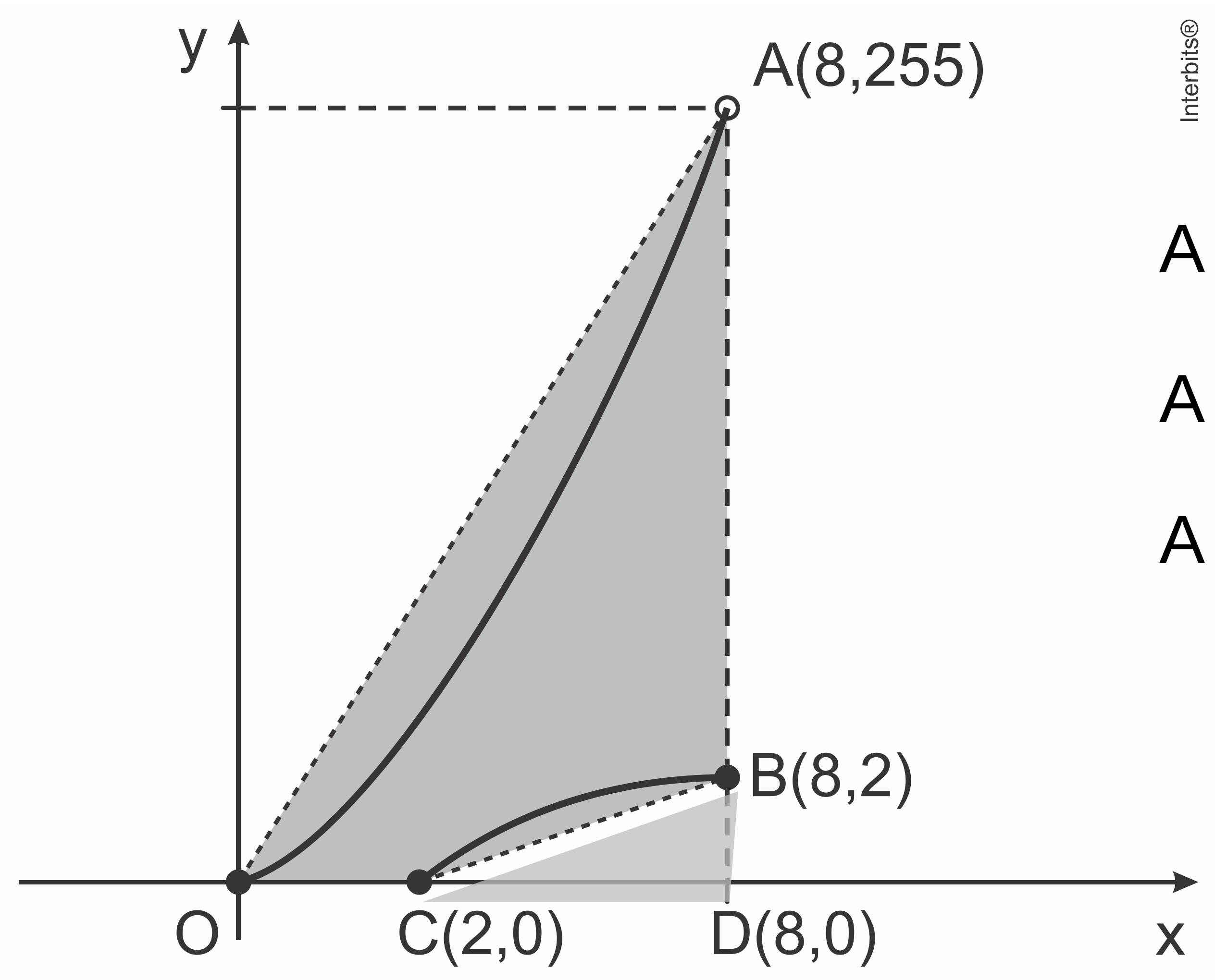


$f(8) = 2^8 - 1 = 255 \Rightarrow$ **A(8, 255)**

$g(8) = \log_2\left(\frac{8}{2}\right) = \log_2 4 = 2 \Rightarrow$ **B(8, 2)**

$g(x) = 0 \Rightarrow \log_2\left(\frac{x}{2}\right) = 0 \Rightarrow \frac{x}{2} = 1 \Rightarrow x = 2 \Rightarrow$ **C(2, 0)**

Portanto, a área pedida será a diferença entre as áreas dos triângulos AOD e BCD. Assim, escrevemos:



$$A = A_{\Delta AOD} - A_{\Delta CDB}$$

$$A = \frac{8 \cdot 255}{2} - \frac{6 \cdot 2}{2}$$

$$A = 1.014$$

C

16)

Seja x o número de metros de tecidos fabricados e vendidos. Daí, devemos ter:

Receita - Despesa > 0

$$4x - (2\,500 + 2,20x) > 0$$

$$4x - 2\,500 - 2,20x > 0$$

$$1,8x > 2\,500$$

$$x > \frac{2\,500}{1,8} \cong 1\,388,89$$

$$x_{mínimo} = 1\,389 \text{ m}$$

B

17) Sejam, respectivamente, x e y o total de quadriláteros convexos e de triângulos que podem ser formados com os pontos dados. Temos:

$$x=\binom{m}{2}\cdot\binom{n}{2}=\frac{m!}{2!\cdot(m-2)!}\cdot\frac{n!}{2!\cdot(n-2)!}=\frac{m\cdot(m-1)}{2}\cdot\frac{n\cdot(n-1)}{2}$$

$$y=m\cdot\binom{n}{2}+n\cdot\binom{m}{2}=m\cdot\frac{n!}{2!\cdot(n-2)!}+n\cdot\frac{m!}{2!\cdot(m-2)!}$$

$$y=\frac{m\cdot n\cdot(n-1)}{2}+\frac{n\cdot m\cdot(m-1)}{2}=\frac{mn}{2}\cdot(m+n-2)$$

$$\frac{x}{y}=\frac{\frac{m\cdot(m-1)}{2}\cdot\frac{n\cdot(n-1)}{2}}{\frac{mn}{2}\cdot(m+n-2)} \Rightarrow \frac{x}{y}=\frac{(m-1)\cdot(n-1)}{2\cdot(m+n-2)}$$

$$\frac{x}{y} = \frac{(m-1)\cdot(n-1)}{2\cdot(m+n-2)}$$

Mas, $\frac{x}{y} = \frac{15}{11}$ e $m+n = 13$, logo,

$$\frac{15}{11} = \frac{(m-1)\cdot(n-1)}{2\cdot 11}$$

$$30 = mn - m - n + 1$$

$$30 = mn - (m+n) + 1$$

$$30 = mn - 13 + 1$$

$$mn = 42$$

A

18)

Temos:
C é o capital aplicado
M = R$ 65.536,00 é o montante

$$M = C.(1+i)^t$$

$$65536 = C \cdot (1{,}01)^4 \cdot (1{,}02)^4 \Leftrightarrow C = \frac{4^8}{1{,}0302^4}$$

$$65536 = 2^{16}$$

$$\Leftrightarrow C = \left(\frac{4}{\sqrt{1{,}0302}}\right)^8$$

$$\Rightarrow C \cong 3{,}94^8.$$

$$\sqrt{1{,}0302} = 1{,}014$$

$$4/1{,}014 = 3{,}944$$

D

19) *O espaço amostral é dado pelo total de pares ordenados em que e são, respectivamente, o ano do século XX em que João nasceu e o ano do século XX em que Maria nasceu. Assim, TEREMOS:*

$$n(\Omega) = 100 \cdot 99$$

$$n(\Omega) = 9\,900$$

O evento A (a soma dos anos em que nasceram é $3\,875$) é fomado por todas as soluções inteiras não negativas da equação $a + b = 3\,875$, com $a = 1901 + \alpha$ e $b = 1901 + \beta$.

$$1901 + \alpha + 1901 + \beta = 3\,875 \Rightarrow \alpha + \beta = 73$$

$$n(A) = P_{74}^{73}$$

$$n(A) = \frac{74!}{73!}$$

$$n(A) = 74$$

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Dessa forma,

$$P(A) = \frac{n(A)}{n(\Omega)}$$

$$P(A) = \frac{74}{9\,900}$$

$$P(A) = \frac{37}{4\,950}$$

C

20)

$$g(x) + a = \frac{\operatorname{sen} x + a^2 + a}{a + 1}$$

$$\textit{Temos que } \; g\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{8}$$

$$g\left(\frac{\pi}{4}\right) + a = \frac{\operatorname{sen}\frac{\pi}{4} + a^2 + a}{a + 1}$$

$$\frac{\sqrt{2}}{8} + a = \frac{\frac{\sqrt{2}}{2} + a^2 + a}{a + 1}$$

$$\frac{\sqrt{2}}{8} \cdot a + a^2 + \frac{\sqrt{2}}{8} + a = \frac{\sqrt{2}}{2} + a^2 + a$$

$$\frac{\sqrt{2}}{8} \cdot a = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{8}$$

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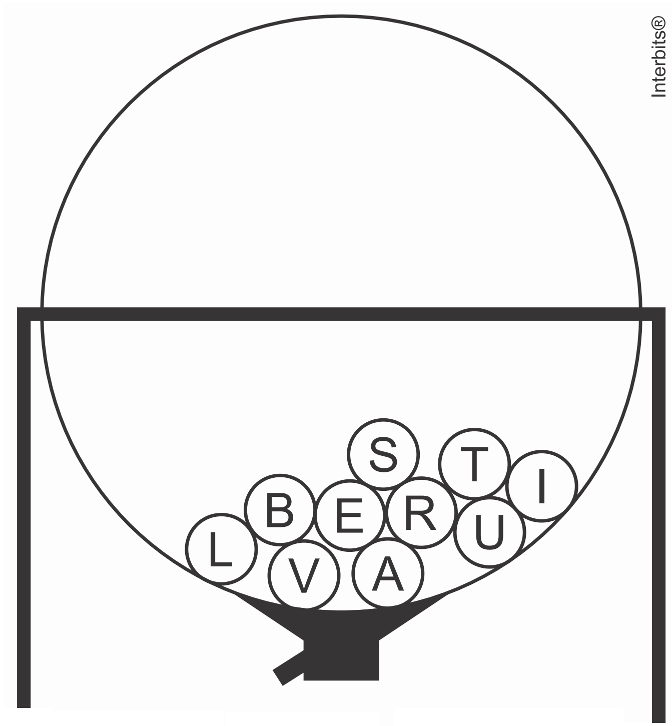
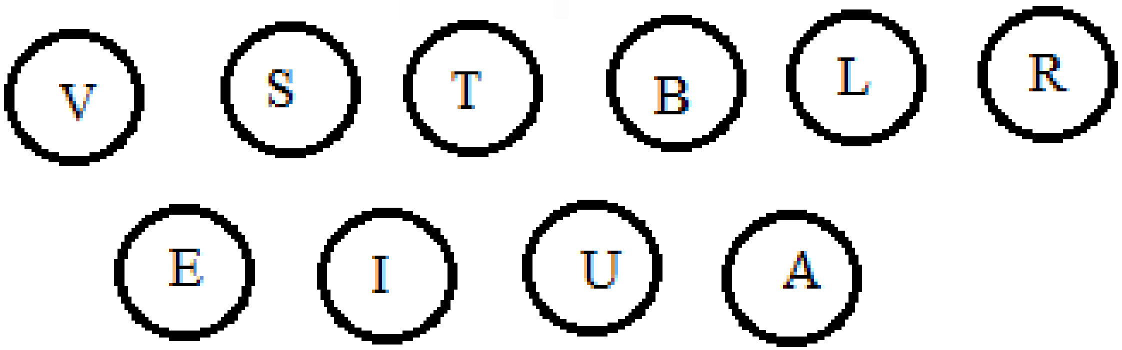
21)



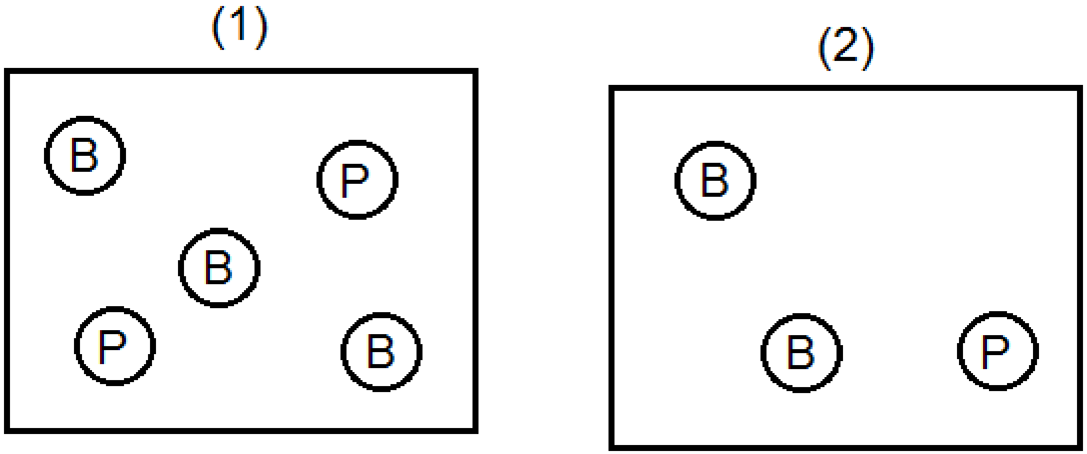
$$p(V \cap V \cap V) = \frac{4}{10} . \frac{3}{9} . \frac{2}{8}$$

$$p(V \cap V \cap V) = \frac{1}{30}$$

C



22)



$$p = p(B_1 \cap B_2) = \frac{3}{5} \cdot \frac{2}{3} = \frac{2}{5}$$

$$p_1 = 1 - p(B_1 \cap B_2) = 1 - \frac{2}{5} \Rightarrow p_1 = \frac{3}{5}$$

$$p_2 = p(B_1 \cap B_2) + p(P_1 \cap P_2)$$

$$p_2 = \frac{2}{5} + \frac{3}{5} \cdot \frac{1}{3} \Rightarrow p_2 = \frac{8}{15}$$

$$\therefore p_1 + p_2 = \frac{3}{5} + \frac{8}{15} = \frac{17}{15}$$

C

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BOA PROVA!